

The Intersection Theorems for New S-KKM Maps and Its Applications

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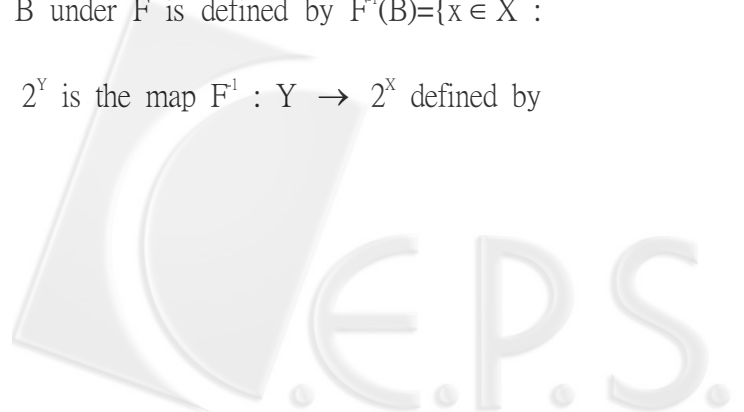
Abstract

In this paper, we shall discuss the intersection theorems on G -convex space for new S -KKM maps. We also establish the Ky Fan type matching theorems. As applications, we discuss the existence results of fixed points for such maps.

Keywords : Convex spaces, Generalized convex spaces, G -convex set, Γ -finite closed, New S -KKM map, New G -KKM map, Matching theorem, Intersection theorems, Fixed points.

1. Introduction and Preliminaries

A set-valued map $F : X \rightarrow 2^Y$ is a mapping from a set X into the power set 2^Y of Y , that is, a mapping with the values $F(x) \subset Y$ for each x in X . For $A \subset X$, $F(A) = \bigcup_{x \in A} F(x)$. For any $B \subset Y$, the inverse of B under F is defined by $F^{-1}(B) = \{x \in X : F(x) \cap B \neq \emptyset\}$. The inverse of $F : X \rightarrow 2^Y$ is the map $F^{-1} : Y \rightarrow 2^X$ defined by



$x \in F^{-1}(y)$ if and only if $y \in F(x)$. Given two maps $F : X \rightarrow 2^Y$ and $G : Y \rightarrow 2^Z$, the composite $GF : X \rightarrow 2^Z$ is defined by $(GF)(x) = G(F(x))$ for all $x \in X$.

In a vector space E , a convex hull of its finite subset will be called a polytope. For topological spaces X , A subset W of X is called compactly closed (compactly open, resp.) if, for any compact set $K \subset X$, $W \cap K$ is closed (open, resp.) in K .

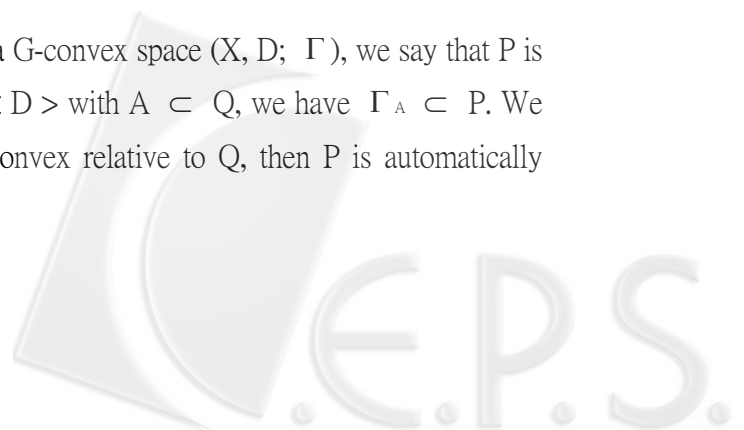
A convex space [3] X is a nonempty convex set (in a vector space) with any topology that induces the Euclidean topology on the convex hulls of its finite sets. We shall denote by $\langle X \rangle$ the family of all non-empty finite subsets of a set X . If X is a subset of a vector space, $\text{co}(X)$ denotes the convex hull of X . For a set A , let $|A|$ denote the cardinality of A . Let Δ^n denote the standard n -simplex $\text{co}\{e_0, \dots, e_n\}$, where e_i is the $(i+1)^{\text{th}}$ unit vector in \mathbb{R}^{n+1} .

A generalized convex space or G -convex space $(X, D; \Gamma)$ [4] consists of a topological space X , a non-empty subset D of X and a map $\Gamma : \langle D \rangle \rightarrow 2^X$ with non-empty values such that

- (1) for each $A, B \in \langle D \rangle$, $A \subset B$ implies $\Gamma(A) \subset \Gamma(B)$; and
- (2) for each $A \in \langle D \rangle$ with $|A| = n+1$, there exists a continuous function $\varphi_A : \Delta^n \rightarrow \Gamma(A)$ such that $J \in \langle A \rangle$ implies $\varphi_A(\Delta^{n-1}) \subset \Gamma(J)$, where Δ^{n-1} denotes the face of Δ^n corresponding to $J \in \langle A \rangle$.

We may write $\Gamma(A) = \Gamma_A$, for each $A \in \langle D \rangle$. If $X=D$, then we may write $(X, X; \Gamma)=(X, \Gamma)$. For a G -convex space $(X, D; \Gamma)$, a subset C of X is said to be G -convex if for each $A \in \langle D \rangle$, $A \subset C$ implies $\Gamma_A \subset C$.

Let P and Q be two non-empty sets in a G -convex space $(X, D; \Gamma)$, we say that P is G -convex relative to Q if for each $A \in \langle D \rangle$ with $A \subset Q$, we have $\Gamma_A \subset P$. We note that if Q is non-empty and P is G -convex relative to Q , then P is automatically



non-empty.

Let X be a nonempty set, $(Y; \Gamma)$ be a G -convex space. The mapping $T: X \rightarrow 2^Y$ is a generalized S-KKM map [2] if for each $N \in \langle X \rangle$, $G\text{-co}(S(N)) \subset T(N)$. The mapping $T: X \rightarrow 2^Y$ is a generalized G-KKM map [5] if for each $N \in \langle X \rangle$, there is a function $s: X \rightarrow Y$ such that $G\text{-co}(s(N)) \subset T(N)$.

Now, we begin to define some new set-valued mappings as follows.

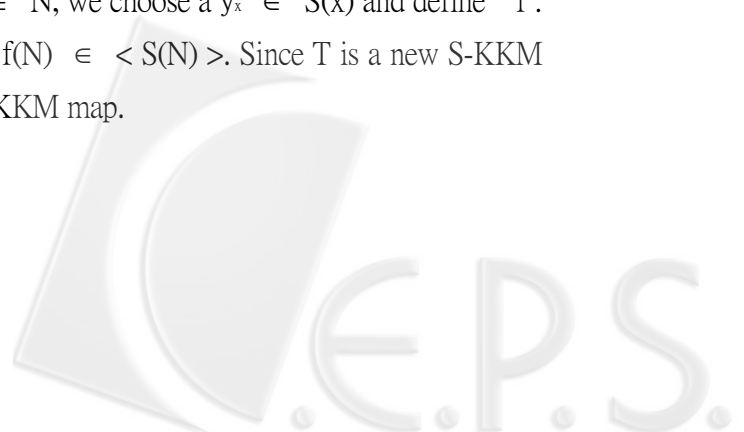
Definition 1. Let X be a nonempty set, $(Y, D; \Gamma)$ be a G -convex space, $S: X \rightarrow 2^D$ with nonempty values, $T: X \rightarrow 2^Y$. We said that T is a new S-KKM map if and only if for each $N \in \langle X \rangle$, $T(N)$ is G -convex relative to $S(N)$.

Definition 2. Let X be a nonempty set, $(Y, D; \Gamma)$ be a G -convex space, $T: X \rightarrow 2^Y$. We said that T is a new G-KKM map if and only if for each $N \in \langle X \rangle$, there is a function $f: N \rightarrow D$ such that $\Gamma_{f(N)} \subset T(N)$ for all $N \in \langle X \rangle$.

From the definition, T is a generalized G-KKM map $\Rightarrow T$ is a generalized S-KKM map $\Rightarrow T$ is a new S-KKM map $\Rightarrow T$ is a new G-KKM map. But converse is not true in general. The following proposition will explain why the last implication hold.

Proposition 1. Let X be a nonempty set, $(Y, D; \Gamma)$ be a G -convex space, $S: X \rightarrow 2^D$ with nonempty values, $T: X \rightarrow 2^Y$. If T is a new S-KKM map, then T is a new G-KKM map.

Proof. For each $N \in \langle X \rangle$. For any $x \in N$, we choose a $y_x \in S(x)$ and define $f: N \rightarrow D$ by $f(x)=y_x$ for all $x \in N$. Then $f(N) \in \langle S(N) \rangle$. Since T is a new S-KKM map, $\Gamma_{f(N)} \subset T(N)$. Hence T is a new G-KKM map.



Definition 3. $(Y, D; \Gamma)$ be a G-convex space and $L \subset Y$, L is said to be Γ -finite closed if for each $A \in \langle D \rangle$, $L \cap \Gamma_A$ is closed in Γ_A .

2. The Intersection Theorems

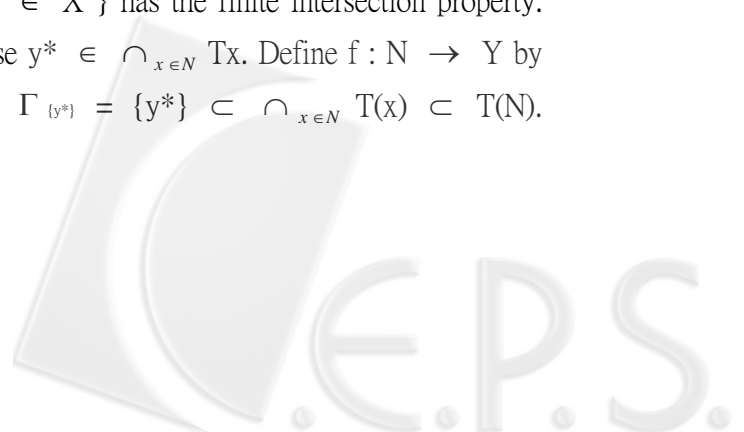
We first state the relation between the finite intersection property and the new G-KKM maps.

Proposition 2. Let X be a nonempty set, $(Y, D; \Gamma)$ be a G-convex space, $T: X \rightarrow 2^Y$ a new G-KKM map with Γ -finite closed values. Then the family $\{T(x) : x \in X\}$ has the finite intersection property.

Proof. Let $N \in \langle X \rangle$, since T is a new G-KKM map, there is a function $f: N \rightarrow D$ such that $\Gamma_{f(N)} \subset T(N)$. Let $L = \Gamma_{f(N)}$. Then there exists a continuous function $\varphi: \Delta^{\text{fr}(N)-1} \rightarrow L$ such that, for all $M \subset N$, $\varphi(\Delta^{\text{fr}(M)-1}) \subset \Gamma_{f(M)} \subset T(M) \cap L$. Hence $\Delta^{\text{fr}(M)-1} \subset \varphi^{-1}(T(M) \cap L) = \bigcup_{x \in M} \varphi^{-1}(T(x) \cap L)$. Note that each $T(x) \cap L$ is closed in L by assumption, so $\varphi^{-1}(T(x) \cap L)$ is closed in $\Delta^{\text{fr}(N)-1}$. By the classical KKM theorem, we have $\bigcap_{x \in N} \varphi^{-1}(T(x) \cap L) \neq \emptyset$ and $\bigcap_{x \in N} (T(x) \cap L) \neq \emptyset$. Hence the family $\{T(x) : x \in X\}$ has the finite intersection property.

Proposition 3. Let X be a nonempty set, $(Y; \Gamma)$ be a G-convex space with $\Gamma_{\{y\}} = \{y\}$ for each $y \in Y$, $T: X \rightarrow 2^Y$. If the family $\{T(x) : x \in X\}$ has the finite intersection property, then T is a new G-KKM map.

Proof. Suppose that the family $\{T(x) : x \in X\}$ has the finite intersection property. Then, for each $N \in \langle X \rangle$, we may choose $y^* \in \bigcap_{x \in N} T(x)$. Define $f: N \rightarrow Y$ by $f(x) = y^*$ for all $x \in N$. Then $\Gamma_{f(N)} = \Gamma_{\{y^*\}} = \{y^*\} \subset \bigcap_{x \in N} T(x) \subset T(N)$. Therefore, T is a new G-KKM map.



Theorem 1. Let X be a nonempty set, $(Y, D; \Gamma)$ be a G -convex space such that, for each $A \in \langle D \rangle$, Γ_A is compact. Suppose that $T : X \rightarrow 2^Y$ with compactly closed values and suppose that there exists a $L \in \langle X \rangle$ such that $\bigcap_{x \in L} T(x)$ is compact.

- (1) If T is a new G -KKM map, then $\bigcap_{x \in X} T(x) \neq \emptyset$.
- (2) If $\bigcap_{x \in X} T(x) \neq \emptyset$ and $\Gamma\{y\}=\{y\}$ for each $y \in Y$, then T is a new G -KKM map.

Proof. (1) Since $T(x)$ is compactly closed, $T(x)$ has Γ -finite closed values. If T is a new G -KKM map, by Proposition 2, the family $\{T(x) : x \in X\}$ has the finite intersection property. Then the family $\{T(x) \cap (\bigcap_{z \in L} T(z)) : x \in X\}$ also has the finite intersection property. Since $\bigcap_{z \in L} T(z)$ is compact and $T(x) \cap (\bigcap_{z \in L} T(z))$ is closed in $\bigcap_{z \in L} T(z)$, we have $\bigcap_{x \in X} T(x) \supseteq \bigcap_{x \in X} (T(x) \cap (\bigcap_{z \in L} T(z))) \neq \emptyset$.

(2) Suppose that $\bigcap_{x \in X} T(x) \neq \emptyset$, then the family $\{T(x) : x \in X\}$ has the finite intersection property. Since $\Gamma\{y\}=\{y\}$, by Proposition 3, T is a new G -KKM map.

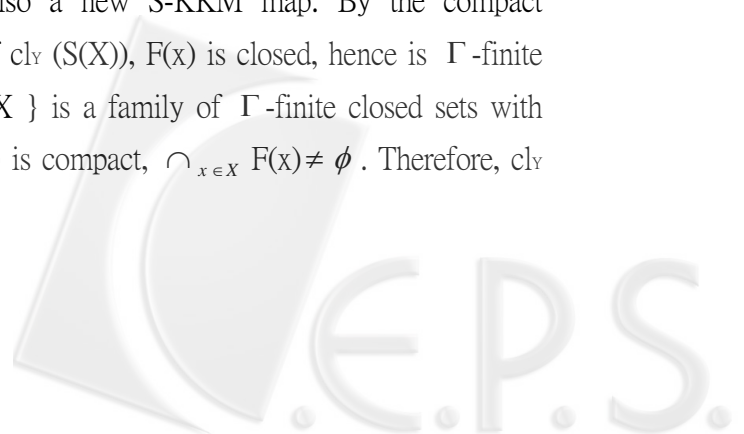
Next, we shall discuss the global intersection property for the new S-KKM map.

Theorem 2. Let X be a nonempty set, $(Y, D; \Gamma)$ be a G -convex space, $T : X \rightarrow 2^Y$, $S : X \rightarrow 2^D$. Suppose that

- (1) for each $x \in X$, $T(x)$ is compactly closed in Y ,
- (2) T is a new S-KKM map,
- (3) $cl_Y(S(X))$ is a compact G -convex set.

Then $cl_Y(S(X)) \cap (\bigcap_{x \in X} T(x)) \neq \emptyset$.

Proof. Since T is a new S-KKM map, for each $N \in \langle X \rangle$ and each $A \in \langle S(N) \rangle$, we have $\Gamma_A \subset T(N)$. Since $cl_Y(S(X))$ is a G -convex set and $\Gamma_A \subset cl_Y(S(X))$, $\Gamma_A \subset cl_Y(S(X)) \cap T(N) = \bigcup_{x \in N} (T(x) \cap cl_Y(S(X))) = \bigcup_{x \in N} F(x) = F(N)$, where $F(x) = T(x) \cap cl_Y(S(X))$. Hence F is also a new S-KKM map. By the compact closedness of $T(x)$ and the compactness of $cl_Y(S(X))$, $F(x)$ is closed, hence is Γ -finite closed. By Proposition 2, $\{F(x) : x \in X\}$ is a family of Γ -finite closed sets with finite intersection property. Since $cl_Y(S(X))$ is compact, $\bigcap_{x \in X} F(x) \neq \emptyset$. Therefore, $cl_Y(S(X)) \cap (\bigcap_{x \in X} T(x)) \neq \emptyset$.



$$(S(X)) \cap (\bigcap_{x \in X} T(x)) \neq \phi .$$

We can deduce the following corollary directly from Theorem 2.

Corollary 1. Let X be a nonempty compact set, $(Y, D; \Gamma)$ be a G -convex space, $T : X \rightarrow 2^Y, S : X \rightarrow 2^D$. Suppose that

- (1) for each $x \in X$, $T(x)$ is compactly closed in Y ,
- (2) T is a new S -KKM map,
- (3) S is upper semi-continuous with compact values and $\text{cl}_Y(S(X))$ is G -convex.

Then $\text{cl}_Y(S(X)) \cap (\bigcap_{x \in X} T(x)) \neq \phi$.

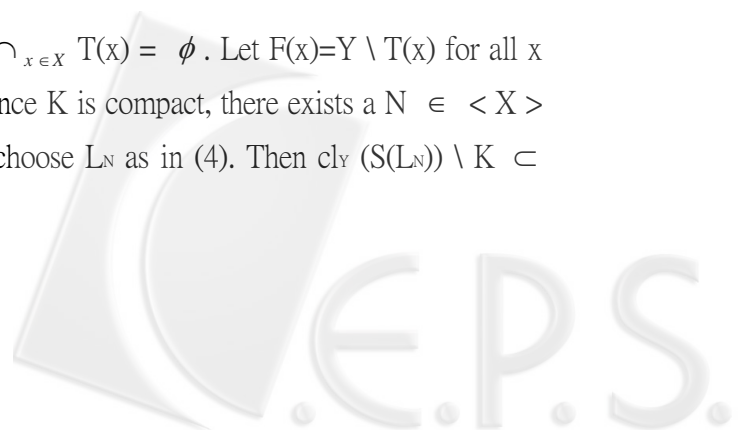
Proof. Since $S : X \rightarrow 2^D$ is upper semi-continuous with compact values and X is compact, $S(X)$ is compact and hence $\text{cl}_Y(S(X))$ is compact. By Theorem 2, $\text{cl}_Y(S(X)) \cap (\bigcap_{x \in X} T(x)) \neq \phi$.

Theorem 3. Let X be a nonempty set in a Hausdorff topological vector space, $(Y, D; \Gamma)$ be a G -convex space, K be a compact subset of Y , $T : X \rightarrow 2^Y, S : X \rightarrow 2^D$. Suppose that

- (1) for each $x \in X$, $T(x)$ is compactly closed in Y ,
- (2) T is a new S -KKM map,
- (3) for each compact convex subset C of X , $\text{cl}_Y(S(C))$ is a compact G -convex subset of Y .
- (4) for each $N \in \langle X \rangle$, there exists a compact convex subset L_N of X containing N such that $\text{cl}_Y(S(L_N)) \cap (\bigcap_{x \in L_N} T(x)) \subset K$.

Then $\text{cl}_Y(S(X)) \cap K \cap (\bigcap_{x \in X} T(x)) \neq \phi$.

Proof. Suppose that $\text{cl}_Y(S(X)) \cap K \cap (\bigcap_{x \in X} T(x)) = \phi$. Let $F(x) = Y \setminus T(x)$ for all $x \in X$. Then $\text{cl}_Y(S(X)) \cap K \subset F(X)$. Since K is compact, there exists a $N \in \langle X \rangle$ such that $\text{cl}_Y(S(X)) \cap K \subset F(N)$. We choose L_N as in (4). Then $\text{cl}_Y(S(L_N)) \setminus K \subset$



$F(L_N)$. Thus,

$$\text{cl}_Y (S(L_N)) \subset F(L_N). \tag{1.1}$$

Since L_N is a compact convex subset in X , $\text{cl}_Y (S(L_N))$ is compact G -convex in Y from (1). Define $H:L_N \rightarrow \text{cl}_Y (S(L_N))$ by $H(x)=T(x) \cap \text{cl}_Y (S(L_N))$ for all $x \in X$. By (2), the map H has closed values in $\text{cl}_Y (S(L_N))$. For each $M \in \langle X \rangle$, by (3), for each $A \in \langle S(M) \rangle$, $\Gamma_A \subset T(M)$. Since $\text{cl}_Y (S(L_N))$ is G -convex, $\Gamma_A \subset \text{cl}_Y (S(L_N))$. Hence $\Gamma_A \subset T(M) \cap \text{cl}_Y (S(L_N))$ and H is a new S-KKM map. By Theorem 2, $\bigcap_{x \in L_N} H(x) \neq \phi$. That is, $\text{cl}_Y (S(L_N)) \cap (\bigcap_{x \in L_N} T(x)) \neq \phi$. Then $\text{cl}_Y (S(L_N)) \not\subset F(L_N)$ which is a contradiction to (1.1). Hence the result follows.

If Y is a convex space and $S(x) = \{ s(x) \}$ for each $x \in X$, where s is a continuous affine mapping, then, for each compact convex subset C of X , $S(C)$ is compact convex subset of Y and so is $\text{cl}_Y (S(C))$. By Theorem 3, we can get the following corollary.

Corollary 2. Let X be a nonempty convex set in a Hausdorff topological vector space, Y be a convex space, K be a compact subset of Y , $T : X \rightarrow 2^Y$, $s : X \rightarrow Y$ be a continuous affine mapping and $S(x) = \{ s(x) \}$ for all $x \in X$. Suppose that

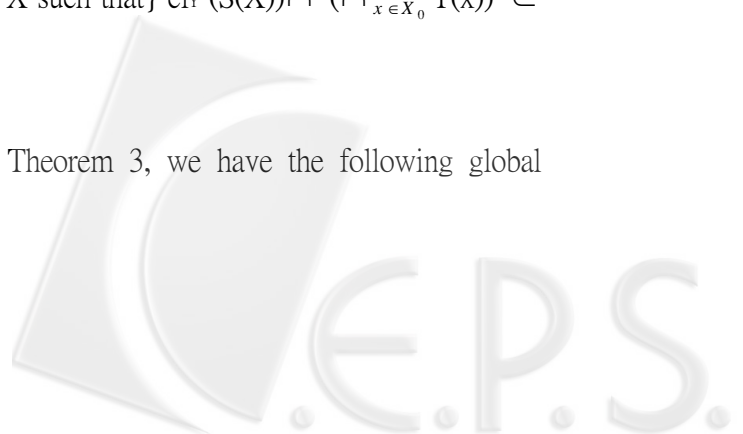
- (1) for each $x \in X$, $T(x)$ is compactly closed in Y ,
- (2) for each $M \in \langle X \rangle$, $\text{co}(S(M)) \subset T(M)$,
- (3) for each $N \in \langle X \rangle$, there exists a compact convex subset L_N of X containing N such that $\text{cl}_Y (S(L_N)) \cap (\bigcap_{x \in L_N} T(x)) \subset K$.

Then $\text{cl}_Y (S(X)) \cap K \cap (\bigcap_{x \in X} T(x)) \neq \phi$.

Remark 1. If we replace the condition (4) in Theorem 3 and the condition (3) in Corollary 2 with the following condition, the conclusions still hold:

there is a compact convex subset X_0 of X such that $\text{cl}_Y (S(X)) \cap (\bigcap_{x \in X_0} T(x)) \subset K$.

Use another condition different from Theorem 3, we have the following global



intersection theorem.

Theorem 4. Let X be a nonempty set, $(Y; \Gamma)$ be a G -convex space, $S, T: X \rightarrow 2^Y$ such that

- (1) T is a new S -KKM map,
- (2) $T: X \rightarrow 2^Y$ has compactly closed values in Y ,
- (3) there exists a compact subset K of Y such that for each $N \in \langle X \rangle$, there exists a compact G -convex subset L_N of Y with $S^{-1}(L_N)$ containing N such that

$$\bigcap_{x \in S^{-1}(L_N)} T(x) \subset K.$$

Then $K \cap (\bigcap_{x \in X} T(x)) \neq \emptyset$.

Proof. By using apagoge and the Brouwer's fixed point theorem, one can easy to deduce the whole intersection is nonempty.

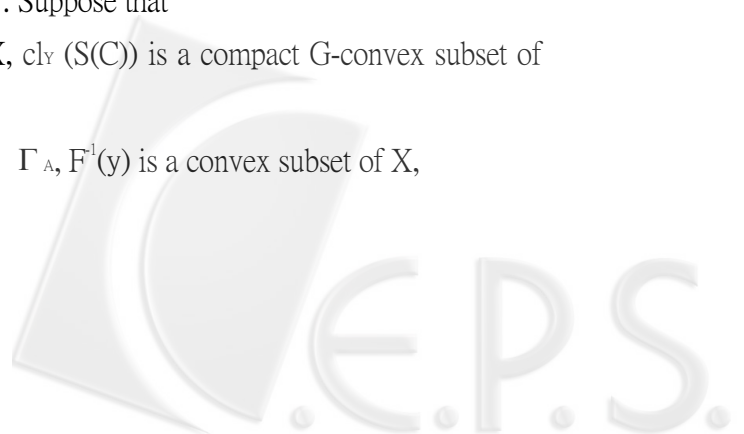
Remark 2. Theorem 4 contains the Theorem 1 in [7].

3. The Existence Results of Fixed Points

We first establish the following matching theorems to deduce the existence result of fixed points.

Theorem 5. Let X be a nonempty set in a Hausdorff topological vector space, (Y, Γ) be a G -convex space, K be a compact subset of Y , $G: X \rightarrow 2^Y$ be a surjection with compactly open values in Y , $S, T: X \rightarrow 2^Y$. Suppose that

- (1) for each compact convex subset C of X , $\text{cl}_Y(S(C))$ is a compact G -convex subset of Y .
- (2) for each $A \in \langle S(X) \rangle$ and each $y \in \Gamma_A$, $F^{-1}(y)$ is a convex subset of X ,



(3) for each $x \in X, G(x) \subset F(x)$,

(4) there exists a compact convex subset X_0 of X such that $\text{cl}_Y (S(X)) \setminus K \subset G(X_0)$.

Then there exist $N \in \langle X \rangle$ and $A \in \langle S(N) \rangle$ such that $\Gamma_A \cap (\bigcap_{x \in \text{co}(N)} F(x)) \neq \phi$.

Proof. From the surjectivity of G and (3), we have $F^{-1}(y) \neq \phi$ for each $A \in \langle S(X) \rangle$ and each $y \in \Gamma_A$. Let $T(x) = Y \setminus G(x)$ for all $x \in X$. Then T has compactly closed values. By (4), $\text{cl}_Y (S(X)) \cap (\bigcap_{x \in X_0} T(x)) \subset K$. Now, we discuss two cases as follows.

Case (a): If $\text{cl}_Y (S(X)) \cap (\bigcap_{x \in X_0} T(x)) = \phi$. Then $\text{cl}_Y (S(X_0)) \cap (\bigcap_{x \in X_0} T(x)) = \phi$. By Theorem 2, T is not a new S-KKM map. Then there exist $N \in \langle X \rangle$ and $A \in \langle S(N) \rangle$ such that $\Gamma_A \not\subset T(N)$. This means that there is a $y \in \Gamma_A$ such that $y \notin T(N) = Y \setminus (\bigcap_{x \in N} G(x))$. Then $y \in \bigcap_{x \in N} G(x) \subset \bigcap_{x \in N} F(x)$, by (3). Thus $N \subset F^{-1}(y)$. From (2), $y \in \bigcap_{x \in \text{co}(N)} F(x)$. That is, $\Gamma_A \cap (\bigcap_{x \in \text{co}(N)} F(x)) \neq \phi$.

Case (b): If $\text{cl}_Y (S(X)) \cap (\bigcap_{x \in X_0} T(x)) \neq \phi$. We claim that T is not a new S-KKM map. Suppose T is a new S-KKM map. Then, by Theorem 3, $\text{cl}_Y (S(X)) \cap K \cap (\bigcap_{x \in X} T(x)) \neq \phi$. Then $\bigcap_{x \in X} T(x) \neq \phi$. Therefore, $G(X) \neq Y$, which contradicts the surjectivity of G . Then, as in the proof of Case (a), we have the conclusion.

Theorem 6. Let $(X; \Gamma)$ be a G -convex space, $f, g, S : X \rightarrow 2^X$ be two set-valued maps such that

- (1) for each $x \in X, f(x)$ is nonempty and G -convex subset of X ,
- (2) g has compactly open values and $f^{-1}(y)$ contains some $g(y)$ for each $y \in X$,
- (3) there is a compact subset K of X such that for each $N \in \langle X \rangle$, there is a compact G -convex subset L_N of X with $S^{-1}(L_N)$ containing N such that

$$\bigcap_{x \in S^{-1}(L_N)} g^c(x) \subset K, \text{ and}$$

- (4) $\bigcup_{x \in X} g(x) = X$.



Then there are $I \in \langle X \rangle$ and $B \in \langle S(I) \rangle$ such that $\Gamma_B \cap \bigcap_{x \in \Gamma_I} f^{-1}(x) \neq \emptyset$.

Proof. Define $T : X \rightarrow 2^X$ by $T(x) = g^c(x)$. From (4), T^c is a surjective map. Hence $\bigcap_{x \in X} T(x) = \emptyset$. By Theorem 4, T is not a new S-KKM map. Then there are $I \in \langle X \rangle$ and $B \in \langle S(I) \rangle$ such that $\Gamma_B \not\subset \bigcup_{x \in I} T(x)$. Thus there is a $z \in \Gamma_B$ such that $z \notin \bigcup_{x \in I} T(x)$ or $z \in \bigcap_{x \in I} T^c(x)$. This implies that $I \subset g^{-1}(z) \subset f(z)$. Since $f(z)$ is G -convex, $\Gamma_I \subset f(z)$ or $z \in \bigcap_{x \in \Gamma_I} f^{-1}(x)$. Therefore, $\Gamma_B \cap \bigcap_{x \in \Gamma_I} f^{-1}(x) \neq \emptyset$.

Corollary 3. Let X be a compact Hausdorff convex space, $G : X \rightarrow 2^X$ be a surjection with compactly open values in X , $S, T : X \rightarrow 2^X$. Suppose that

- (1) for each compact convex subset C of X , $\text{cl}_X(S(C))$ is a compact convex subset of X .
- (2) for each $y \in \text{co}(S(X))$, $F^{-1}(y)$ is a convex subset of X ,
- (3) for each $x \in X$, $G(x) \subset F(x)$,

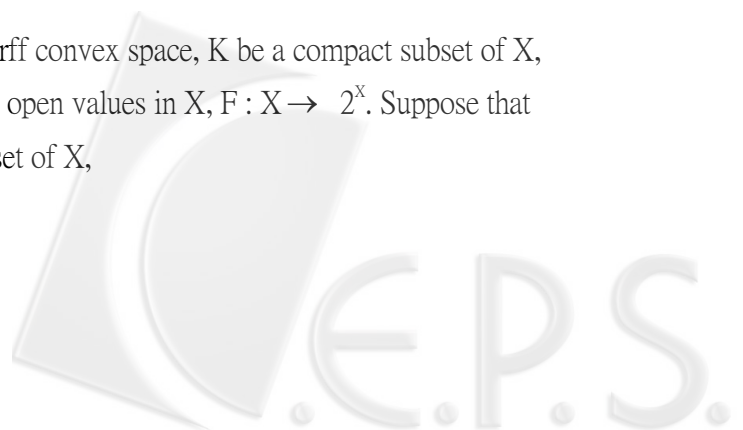
Then there exist $N \in \langle X \rangle$ and $A \in \langle S(N) \rangle$ such that $\text{co}(A) \cap (\bigcap_{x \in \text{co}(N)} F(x)) \neq \emptyset$.

Proof. Since X is compact and G is surjective with compactly open values, there exists an $N \in \langle X \rangle$ such that $X \subset G(N)$. Hence $\text{cl}_X S(X) \subset X \subset G(N) \subset G(\text{co}(N))$. If we take $X_0 = \text{co}(N)$ and $K = \emptyset$, then the condition (4) of Theorem 4 holds. Applying Theorem 4, we have the conclusion.

If we take $S(x) = \{x\}$ for each $x \in X$ and $X=Y$ is a convex space in Theorem 4, then we have the following fixed point theorem.

Corollary 4. Let X be a nonempty Hausdorff convex space, K be a compact subset of X , $G : X \rightarrow 2^X$ be a surjection with compactly open values in X , $F : X \rightarrow 2^X$. Suppose that

- (1) for each $y \in X$, $F^{-1}(y)$ is a convex subset of X ,



(2) for each $x \in X$, $G(x) \subset F(x)$,

(3) there exists a compact convex subset X_0 of X such that $X \setminus K \subset G(X_0)$.

Then there exists an $N \in \langle X \rangle$ such that $\text{co}(N) \cap (\bigcap_{x \in \text{co}(N)} F(x)) \neq \emptyset$. Hence, there exists a $\bar{y} \in \text{co}(N)$ such that $\bar{y} \in F(\bar{y})$.

Remark 3. Corollary 4 generalizes Tarafdar's fixed point theorem [6].

Corollary 5. Let X be a nonempty Hausdorff compact convex space, $G: X \rightarrow 2^X$ be a surjection with compactly open values in X , $F: X \rightarrow 2^X$. Suppose that

(1) for each $y \in X$, $F^1(y)$ is a convex subset of X ,

(2) for each $x \in X$, $G(x) \subset F(x)$.

Then there exists $N \in \langle X \rangle$ such that $\text{co}(N) \cap (\bigcap_{x \in \text{co}(N)} F(x)) \neq \emptyset$. In particular, there exists $\bar{y} \in \text{co}(N)$ such that $\bar{y} \in F(\bar{y})$. That is, F has a fixed point \bar{y} in $\text{co}(N)$.

Remark 4. Corollary 5 generalizes Browder's fixed point theorem [1].

To the end, we can easy to derive the following fixed point theorem form Theorem 6 with S is an identity map.

Corollary 6. Let $(X; \Gamma)$ be a G -convex space, $f, g, S : X \rightarrow 2^X$ be two set-valued maps such that

(1) for each $x \in X$, $f(x)$ is nonempty and G -convex subset of X ,

(2) g has compactly open values and $f^1(y)$ contains some $g(y)$ for each $y \in X$,

(3) there is a compact subset K of X such that for each $N \in \langle X \rangle$, there is a compact

G -convex subset L_N of X with L_N containing N such that $\bigcap_{x \in L_N} g^c(x) \subset K$, and

(4) $\bigcup_{x \in X} g(x) = X$.



Then there is a $B \in \langle X \rangle$ such that $\Gamma_B \cap \bigcap_{x \in \Gamma_B} f^{-1}(x) \neq \emptyset$. In particular, f has a fixed point.

References

- [1] F. E. Browder, 'On nonlinear monotone operators and convex sets in Banach space', Bull. Amer. Math. Soc. 71(1965), 780-785.
- [2] L. J. Lin and T. H. Chang, 'S-KKM theorems, saddle points and minimax inequalities', Nonlinear Analysis, 34 (1998), 73-86.
- [3] S. Park, 'Generalizations of Ky Fan's matching theorems and their applications', J. Math. Anal. Appl., 141 (1989), 164-176.
- [4] S. Park and H. Kim, 'Admissible classes of multifunctions on generalized convex spaces', Proc. Coll. Natur. Sci. Seoul National University, 18 (1993), 1-21.
- [5] K. K. Tan, 'G-KKM Theorem, minimax inequalities and saddle points', Nonlinear Anal. Theory, Meth. Appl., 30 (1997), 4151-4160.
- [6] E. Tarafdar, 'A fixed theorem equivalent to the Fan-Knaster-Kuratowski-Mazurkiewicz theorem', J. Math. Anal. Appl., 128 (1987), 475-479.
- [7] E. Tarafdar, 'Fixed point theorems in H-space and equilibrium points of abstract economics', J. Austral. Math. Soc. (Series A), 53 (1992), 252-260.



新 S-KKM 映射的全交集定理及其應用

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摘 要

交集理論在非線性分析中，被廣泛討論及運用著。本篇文章裡，我們將在 G-凸空間裡討論新 S-KKM 映射的交集理論。我們也討論了樊璣型的配對理論，並討論集合值映射之定點的存在性作為它的應用。

關鍵字：凸空間，G-凸空間，G-凸集， Γ -有限閉集，新 S-KKM 映射，新 G-KKM 映射，配對理論，交集理論，定點。

