The Intersection Theorems for New S-KKM Maps and Its Applications

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Abstract

In this paper, we shall discuss the intersection theorems on G-convex spa ce for new S-KKM maps. We also establish the Ky Fan type matching theor ems. As applications, we discuss the existence results of fixed points for suc h maps.

Keywords： Convex spaces, Generalized convex spaces, G-convex set, Γ-finite closed, New S-KKM map, New G-KKM map, Matching theorem, Intersection theorems, Fixed points.

1. Introduction and Preliminaries

A set-valued map $F : X \to 2^Y$ is a mapping from a set X into the power set 2^Y of Y, that is, a mapping with the values $F(x) \subset Y$ for each x in X. For $A \subset X$, $F(A) = \bigcup_{x \in A} F(x)$. For any $B \subset Y$, the inverse of B under F is defined by $F^{-1}(B)=\{x \in X : F(x) \cap B \neq \phi\}$. The inverse of $F : X \to 2^Y$ is the map $F^{-1} : Y \to 2^X$ defined by

$x \in F^{-1}(y)$ if and only if $y \in F(x)$. Given two maps $F : X \to 2^Y$ and $G : Y \to 2^Z$, the composite $GF : X \to 2^Z$ is defined by $(GF)(x) = G(F(x))$ for all $x \in X$.

In a vector space E, a convex hull of its finite subset will be called a polytope. For topological spaces X, A subset W of X is called compactly closed (compactly open, resp.) if, for any compact set $K \subset X$, $W \cap K$ is closed (open, resp.) in K.

A convex space [3] X is a nonempty convex set (in a vector space) with any topology that induces the Euclidean topology on the convex hulls of its finite sets. We shall denote by <X> the family of all non-empty finite subsets of a set X. If X is a subset of a vector space, co(X) denotes the convex hull of X. For a set A, let |A| denote the cardinality of A. Let Δ^n denote the standard n-simplex $co\{e_0, \ldots, e_n\}$, where e_i is the $(i+1)^{th}$ unit vector in R^{n+1}.

A generalized convex space or G-convex space $(X, D; \Gamma)$ [4] consists of a topological space X, a non-empty subset D of X and a map $\Gamma : \langle D \rangle \to 2^X$ with non-empty values such that

(1) for each $A, B \in \langle D \rangle$, $A \subset B$ implies $\Gamma(A) \subset \Gamma(B)$; and

(2) for each $A \in \langle D \rangle$ with $|A| = n+1$, there exists a continuous function $\varphi_A : \Delta^n \to \Gamma(A)$ such that $J \in \langle A \rangle$ implies $\varphi_A(\Delta^{|J|-1}) \subset \Gamma(J)$, where $\Delta^{|J|-1}$ denotes the face of Δ^n corresponding to $J \in \langle A \rangle$.

We may write $\Gamma(A) = \Gamma_A$, for each $A \in \langle D \rangle$. If X=D, then we may write $(X, X; \Gamma)=(X, \Gamma)$. For a G-convex space $(X, D; \Gamma)$, a subset C of X is said to be G-convex if for each $A \in \langle D \rangle$, $A \subset C$ implies $\Gamma_A \subset C$.

Let P and Q be two non-empty sets in a G-convex space $(X, D; \Gamma)$, we say that P is G-convex relative to Q if for each $A \in \langle D \rangle$ with $A \subset Q$, we have $\Gamma_A \subset P$. We note that if Q is non-empty and P is G-convex relative to Q, then P is automatically non-empty.

Let X be a nonempty set, (Y; Γ) be a G-convex space. The mapping T: X $\rightarrow 2^Y$ is a generalized S-KKM map [2] if for each N $\in$ < X >, G-co(S(N)) $\subset$ T(N). The mapping T: X $\rightarrow 2^Y$ is a generalized G-KKM map [5] if for eac h N $\in$ < X >, there is a function s: X $\rightarrow$ Y such that G-co(s(N)) $\subset$ T(N).

Now, we begin to define some new set-valued mappings as follows.

Definition 1. Let X be a nonempty set, (Y, D; Γ) be a G-convex space, S: X $\rightarrow 2^D$ with nonempty values, T: X $\rightarrow 2^Y$. We said that T is a new S-KKM map if and only if for each N $\in$ < X >, T(N) is G-convex relative to S(N).

Definition 2. Let X be a nonempty set, (Y, D; Γ) be a G-convex space, T: X $\rightarrow 2^Y$. We said that T is a new G-KKM map if and only if for each N $\in$ < X >, there is a function f : N $\rightarrow$ D such that $\Gamma_{f(M)} \subset$ T(M) for all M $\in$ < N >.

From the definition, T is a generalized G-KKM map $\Rightarrow$ T is a generalized S-KKM map $\Rightarrow$ T is a new S-KKM map $\Rightarrow$ T is a new G-KKM map. But converse is not true in general. The following proposition will explain why the last implication hold.

Proposition 1. Let X be a nonempty set, (Y, D; Γ) be a G-convex space, S: X $\rightarrow 2^D$ with nonempty values, T : X $\rightarrow 2^Y$. If T is a new S-KKM map, then T is a new G-KKM map.

Proof. For each N $\in$ < X >. For any x $\in$ N, we choose a $y_x \in$ S(x) and define f : N $\rightarrow$ D by f(x)=y_x for all x $\in$ N. Then f(N) $\in$ < S(N) >. Since T is a new S-KKM map, $\Gamma_{f(N)} \subset$ T(N). Hence T is a new G-KKM map.

Definition 3. $(Y, D; \Gamma)$ be a G-convex space and $L \subset Y$, L is said to be Γ-finite closed if for each $A \in \langle D \rangle$, $L \cap \Gamma_A$ is closed in Γ_A.

2. The Intersection Theorems

We first state the relation between the finite intersection property and the new G-KKM maps.

Proposition 2. Let X be a nonempty set, $(Y, D; \Gamma)$ be a G-convex space, $T: X \to 2^Y$ a new G-KKM map with Γ-finite closed values. Then the family $\{ T(x) : x \in X \}$ has the finite intersection property.

Proof. Let $N \in \langle X \rangle$, since T is an new G-KKM map, there is a function $f : N \to D$ such that $\Gamma_{f(N)} \subset T(N)$. Let $L = \Gamma_{f(N)}$. Then there exists a continuous function $\varphi : \Delta^{|f(N)|-1} \to L$ such that, for all $M \subset N$, $\varphi(\Delta^{|f(M)|-1}) \subset \Gamma_{f(M)} \subset T(M) \cap L$. Hence $\Delta^{|f(M)|-1} \subset \varphi^{-1}(T(M) \cap L) = \cup_{x \in M} \varphi^{-1}(T(x) \cap L)$. Note that each $T(x) \cap L$ is closed in L by assumption, so $\varphi^{-1}(T(x) \cap L)$ is closed in $\Delta^{|f(N)|-1}$. By the classical KKM theorem, we have $\cap_{x \in N} \varphi^{-1}(T(x) \cap L) \neq \phi$ and $\cap_{x \in N} (T(x) \cap L) \neq \phi$. Hence the family $\{T(x) : x \in X\}$ has the finite intersection property.

Proposition 3. Let X be a nonempty set, $(Y; \Gamma)$ be a G-convex space with $\Gamma_{\{y\}} = \{ y \}$ for each $y \in Y$, $T : X \to 2^Y$. If the family $\{T(x) : x \in X\}$ has the finite intersection property, then T is a new G-KKM map.

Proof. Suppose that the family $\{ T(x) : x \in X \}$ has the finite intersection property. Then, for each $N \in \langle X \rangle$, we may choose $y^* \in \cap_{x \in N} Tx$. Define $f : N \to Y$ by $f(x) = y^*$ for all $x \in N$. Then $\Gamma_{f(N)} = \Gamma_{\{y^*\}} = \{y^*\} \subset \cap_{x \in N} T(x) \subset T(N)$. Therefore, T is a new G-KKM map.

Theorem 1. Let X be a nonempty set, (Y, D; Γ) be a G-convex space such that, for each A $\in$ < D >, Γ_A is compact. Suppose that T : X $\to$ 2^Y with compactly closed values and suppose that there exists a L $\in$ < X > such that $\cap_{x\in L}$ T(z) is compact.

(1) If T is a new G-KKM map, then $\cap_{x\in X}$ T(x) $\neq \phi$.

(2) If $\cap_{x\in X}$ T(x) $\neq \phi$ and Γ {y}={y} for each y $\in$ Y, then T is a new G-KKM map.

Proof. (1) Since T(x) is compactly closed, T(x) has Γ-finite closed values. If T is a new G-KKM map, by Proposition 2, the family { T(x) : x $\in$ X } has the finite intersection property. Then the family { T(x) $\cap$ ($\cap_{z\in L}$ T(z)) : x $\in$ X } also has the finite intersection property. Since $\cap_{z\in L}$ T(z) is compact and T(x) $\cap$ ($\cap_{z\in L}$ T(z)) is closed in $\cap_{z\in L}$ T(z), we have $\cap_{x\in X}$ T(x) $\supset$ $\cap_{x\in X}$ (T(x) $\cap$($\cap_{z\in L}$ T(z))) $\neq \phi$.

(2) Suppose that $\cap_{x\in X}$ T(x) $\neq \phi$, then the family { T(x) : x $\in$ X } has the finite intersection property. Since Γ {y}={y}, by Proposition 3, T is a new G-KKM map.

Next, we shall discuss the global intersection property for the new S-KKM map.

Theorem 2. Let X be a nonempty set, (Y, D; Γ) be a G-convex space, T : X $\to$ 2^Y, S : X$\to$ 2^D. Suppose that

(1) for each x $\in$ X, T(x) is compactly closed in Y,

(2) T is a new S-KKM map,

(3) cl_Y(S(X)) is a compact G-convex set.

Then cl_Y(S(X)) $\cap$ ($\cap_{x\in X}$ T(x)) $\neq \phi$.

Proof. Since T is a new S-KKM map, for each N $\in$ < X > and each A $\in$ < S(N) >, we have Γ_A $\subset$ T(N). Since cl_Y (S(X)) is a G-convex set and Γ_A $\subset$$cl_Y$ (S(X)), Γ_A $\subset$ cl_Y (S(X)) $\cap$ T(N)= $\cup_{x\in N}$ (T(x) $\cap$ cl_Y (S(X))) =$\cup_{x\in N}$ F(x)=F(N), where F(x)=T(x) $\cap$ cl_Y (S(X)). Hence F is also a new S-KKM map. By the compact closedness of T(x) and the compactness of cl_Y (S(X)), F(x) is closed, hence is Γ-finite closed. By Proposition 2, { F(x) : x $\in$ X } is a family of Γ-finite closed sets with finite intersection property. Since cl_Y(S(X)) is compact, $\cap_{x\in X}$ F(x)$\neq \phi$. Therefore, cl_Y

$(S(X)) \cap (\cap_{x \in X} T(x)) \neq \phi$.

We can deduce the following corollary directly from Theorem 2.

Corollary 1. Let X be a nonempty compact set, (Y, D; Γ) be a G-convex space, T : X $\to 2^Y$, S : X $\to 2^D$. Suppose that
(1) for each x $\in$ X, T(x) is compactly closed in Y,
(2) T is a new S-KKM map,
(3) S is upper semi-continuous with compact values and cl_Y (S(X)) is G-convex.
Then $cl_Y (S(X)) \cap (\cap_{x \in X} T(x)) \neq \phi$.

Proof. Since S : X $\to 2^D$ is upper semi-continuous with compact values and X is compact, S(X) is compact and hence cl_Y (S(X)) is compact. By Theorem 2, $cl_Y (S(X)) \cap (\cap_{x \in X} T(x)) \neq \phi$.

Theorem 3. Let X be a nonempty set in a Hausdorff topological vector space, (Y, D; Γ) be a G-convex space, K be a compact subset of Y, T : X $\to 2^Y$, S : X $\to 2^D$. Suppose that
(1) for each x $\in$ X, T(x) is compactly closed in Y,
(2) T is a new S-KKM map,
(3) for each compact convex subset C of X, cl_Y (S(C)) is a compact G-convex subset of Y.
(4) for each N $\in$ < X >, there exists a compact convex subset L_N of X containing N such that $cl_Y (S(L_N)) \cap (\cap_{x \in L_N} T(x)) \subset K$.
Then $cl_Y (S(X)) \cap K \cap (\cap_{x \in X} T(x)) \neq \phi$.

Proof. Suppose that $cl_Y (S(X)) \cap K \cap (\cap_{x \in X} T(x) = \phi$. Let F(x)=Y \ T(x) for all x $\in$ X. Then $cl_Y (S(X)) \cap K \subset F(X)$. Since K is compact, there exists a N $\in$ < X > such that $cl_Y (S(X)) \cap K \subset F(N)$. We choose L_N as in (4). Then $cl_Y (S(L_N)) \setminus K \subset$

$F(L_N)$. Thus,

$$cl_Y\ (S(L_N)) \subset F(L_N). \tag{1.1}$$

Since L_N is a compact convex subset in X, cl_Y $(S(L_N))$ is compact G-convex in Y from (1). Define $H{:}L_N \rightarrow cl_Y\ (S(L_N))$ by $H(x)=T(x) \cap cl_Y\ (S(L_N))$ for all $x \in X$. By (2), the map H has closed values in cl_Y $(S(L_N))$. For each $M \in \langle X \rangle$, by (3), for each $A \in \langle S(M) \rangle$, $\Gamma_A \subset T(M)$. Since cl_Y $(S(L_N))$ is G-convex, $\Gamma_A \subset cl_Y\ (S(L_N))$. Hence $\Gamma_A \subset T(M) \cap cl_Y\ (S(L_N))$ and H is a new S-KKM map. By Theorem 2, $\cap_{x \in L_N} H(x) \neq \phi$. That is, $cl_Y\ (S(L_N)) \cap (\cap_{x \in L_N} T(x)) \neq \phi$. Then $cl_Y\ (S(L_N)) \not\subset F(L_N)$ which is a contradiction to (1.1). Hence the result follows.

If Y is a convex space and $S(x) = \{ s(x) \}$ for each $x \in X$, where s is a continuous affine mapping, then, for each compact convex subset C of X, S(C) is compact convex subset of Y and so is cl_Y (S(C)). By Theorem 3, we can get the following corollary.

Corollary 2. Let X be a nonempty convex set in a Hausdorff topological vector space, Y be a convex space, K be a compact subset of Y, $T : X \rightarrow 2^Y$, $s : X \rightarrow Y$ be a continuous affine mapping and $S(x) = \{ s(x) \}$ for all $x \in X$. Suppose that

(1) for each $x \in X$, T(x) is compactly closed in Y,

(2) for each $M \in \langle X \rangle$, $co(S(M)) \subset T(M)$,

(3) for each $N \in \langle X \rangle$, there exists a compact convex subset L_N of X containing N such that $cl_Y\ (S(L_N)) \cap (\cap_{x \in L_N} T(x)) \subset K$.

Then $cl_Y\ (S(X)) \cap K \cap (\cap_{x \in X} T(x)) \neq \phi$.

Remark 1. If we replace the condition (4) in Theorem 3 and the condition (3) in Corollary 2 with the following condition, the conclusions still hold:

there is a compact convex subset X_0 of X such that} $cl_Y\ (S(X)) \cap (\cap_{x \in X_0} T(x)) \subset K$.

Use another condition different from Theorem 3, we have the following global intersection theorem.

Theorem 4. Let X be a nonempty set, (Y; Γ) be a G-convex space, S, T: X $\to$ 2^Y such that

(1) T is a new S-KKM map,

(2) T : X $\to$ 2^Y has compactly closed values in Y,

(3) there exists a compact subset K of Y such that for each N $\in$ < X >, there exists a compact G-convex subset L_N of Y with $S^{-1}(L_N)$ containing N such that $\cap_{x \in S^{-1}(L_N)} T(x) \subset K$.

Then $K \cap (\cap_{x \in X} T(x)) \neq \phi$.

Proof. By using apagoge and the Brouwer's fixed point theorem, one can easy to deduce the whole intersection is nonempty.

Remark 2. Theorem 4 contains the Theorem 1 in [7].

3. The Existence Results of Fixed Points

We first establish the following matching theorems to deduce the existence result of fixed points.

Theorem 5. Let X be a nonempty set in a Hausdorff topological vector space, (Y, Γ) be a G-convex space, K be a compact subset of Y, G : X $\to$ 2^Y be a surjection with compactly open values in Y, S, T : X$\to$ 2^Y. Suppose that

(1) for each compact convex subset C of X, cl_Y (S(C)) is a compact G-convex subset of Y.

(2) for each A $\in$ < S(X) > and each y $\in$ Γ_A, $F^{-1}(y)$ is a convex subset of X,

(3) for each $x \in X$, $G(x) \subset F(x)$,

(4) there exists a compact convex subset X_0 of X such that $cl_Y(S(X)) \setminus K \subset G(X_0)$.

Then there exist $N \in \langle X \rangle$ and $A \in \langle S(N) \rangle$ such that $\Gamma_A \cap (\cap_{x \in co(N)} F(x)) \neq \phi$.

Proof. From the surjectivity of G and (3), we have $F^{-1}(y) \neq \phi$ for each $A \in \langle S(X) \rangle$ and each $y \in \Gamma_A$. Let $T(x)=Y \setminus G(x)$ for all $x \in X$. Then T has compactly closed values. By (4), $cl_Y(S(X)) \cap (\cap_{x \in X_0} T(x)) \subset K$. Now, we discuss two cases as follows.

Case (a): If $cl_Y(S(X)) \cap (\cap_{x \in X_0} T(x)) = \phi$. Then $cl_Y(S(X_0)) \cap (\cap_{x \in X_0} T(x)) = \phi$. By Theorem 2, T is not a new S-KKM map. Then there exist $N \in \langle X \rangle$ and $A \in \langle S(N) \rangle$ such that $\Gamma_A \not\subset T(N)$. This means that there is a $y \in \Gamma_A$ such that $y \notin T(N)=Y \setminus (\cap_{x \in N} G(x))$. Then $y \in \cap_{x \in N} G(x) \subset \cap_{x \in N} F(x)$, by (3). Thus $N \subset F^{-1}(y)$. From (2), $y \in \cap_{x \in co(N)} F(x))$. That is, $\Gamma_A \cap (\cap_{x \in co(N)} F(x)) \neq \phi$.

Case (b): If $cl_Y(S(X)) \cap (\cap_{x \in X_0} T(x)) \neq \phi$. We claim that T is not a new S-KKM map. Suppose T is a new S-KKM map. Then, by Theorem 3, $cl_Y(S(X)) \cap K \cap (\cap_{x \in X} T(x)) \neq \phi$. Then $\cap_{x \in X} T(x) \neq \phi$. Therefore, $G(X) \neq Y$, which contradicts the surjectivity of G. Then, as in the proof of Case (a), we have the conclusion.

Theorem 6. Let $(X; \Gamma)$ be a G-convex space, f, g, S : $X \rightarrow 2^X$ be two set-valued maps such that

(1) for each $x \in X$, $f(x)$ is nonempty and G-convex subset of X,

(2) g has compactly open values and $f^{-1}(y)$ contains some $g(y)$ for each $y \in X$,

(3) there is a compact subset K of X such that for each $N \in \langle X \rangle$, there is a co mpact G-convex subset L_N of X with $S^{-1}(L_N)$ containing N such that

$$\bigcap_{x \in S^{-1}(L_N)} g^c(x) \subset K, \text{ and}$$

(4) $\bigcup_{x \in X} g(x) = X$.

Then there are $I \in \langle X \rangle$ and $B \in \langle S(I) \rangle$ such that $\Gamma_B \cap \bigcap_{x \in \Gamma_I} f^{-1}(x) \neq \phi$.

Proof. Define $T : X \rightarrow 2^X$ by $T(x) = g^c(x)$. From (4), T^c is a surjective map. Hence $\bigcap_{x \in X} T(x) = \phi$. By Theorem 4, T is not a new S-KKM map. Then there are $I \in \langle X \rangle$ and $B \in \langle S(I) \rangle$ such that $\Gamma_B \not\subset \bigcup_{x \in I} T(x)$. Thus there is a $z \in \Gamma_B$ such that $z \notin \bigcup_{x \in I} T(x)$ or $z \in \bigcap_{x \in I} T^c(x)$. This implies that $I \subset g^{-1}(z) \subset f(z)$. Since f(z) is G-convex, $\Gamma_I \subset f(z)$ or $z \in \bigcap_{x \in \Gamma_I} f^{-1}(x)$. Therefore, $\Gamma_B \cap \bigcap_{x \in \Gamma_I} f^{-1}(x) \neq \phi$.

Corollary 3. Let X be a compact Hausdorff convex space, $G : X \rightarrow 2^X$ be a surjection with compactly open values in X, S, $T : X \rightarrow 2^X$. Suppose that

(1) for each compact convex subset C of X, $cl_X(S(C))$ is a compact convex subset of X.

(2) for each $y \in co(S(X))$, $F^{-1}(y)$ is a convex subset of X,

(3) for each $x \in X$, $G(x) \subset F(x)$,

Then there exist $N \in \langle X \rangle$ and $A \in \langle S(N) \rangle$ such that $co(A) \cap (\bigcap_{x \in co(N)} F(x)) \neq \phi$.

Proof. Since X is compact and G is surjective with compactly open values, there exists an $N \in \langle X \rangle$ such that $X \subset G(N)$. Hence $cl_Y S((X)) \subset X \subset G(N) \subset G(co(N))$. If we take $X_0 = co(N)$ and $K = \phi$, then the condition (4) of Theorem 4 holds. Applying Theorem 4, we have the conclusion.

If we take $S(x) = \{ x \}$ for each $x \in X$ and X=Y is a convex space in Theorem 4, then we have the following fixed point theorem.

Corollary 4. Let X be a nonempty Hausdorff convex space, K be a compact subset of X, $G : X \rightarrow 2^X$ be a surjection with compactly open values in X, $F : X \rightarrow 2^X$. Suppose that

(1) for each $y \in X$, $F^{-1}(y)$ is a convex subset of X,

(2) for each $x \in X$, $G(x) \subset F(x)$,

(3) there exists a compact convex subset X_0 of X such that $X \setminus K \subset G(X_0)$.

Then there exists an $N \in \langle X \rangle$ such that $co(N) \cap (\cap_{x \in co(N)} F(x)) \neq \phi$. Hence, there exists a $\bar{y} \in co(N)$ such that $\bar{y} \in F(\bar{y})$.

Remark 3. Corollary 4 generalizes Tarafdar's fixed point theorem [6].

Corollary 5. Let X be a nonempty Hausdorff compact convex space, $G: X \to 2^X$ be a surjection with compactly open values in X, $F: X \to 2^X$. Suppose that

(1) for each $y \in X$, $F^{-1}(y)$ is a convex subset of X,

(2) for each $x \in X$, $G(x) \subset F(x)$.

Then there exists $N \in \langle X \rangle$ such that $co(N) \cap (\cap_{x \in co(N)} F(x)) \neq \phi$. In particular, there exists $\bar{y} \in co(N)$ such that $\bar{y} \in F(\bar{y})$. That is, F has a fixed point $\bar{y}$ in co(N).

Remark 4. Corollary 5 generalizes Browder's fixed point theorem [1].

To the end, we can easy to derive the following fixed point theorem form Theorem 6 with S is an identity map.

Corollary 6. Let $(X; \Gamma)$ be a G-convex space, f, g, $S : X \to 2^X$ be two set-valued maps such that

(1) for each $x \in X$, f(x) is nonempty and G-convex subset of X,

(2) g has compactly open values and $f^{-1}(y)$ contains some g(y) for each $y \in X$,

(3) there is a compact subset K of X such that for each $N \in \langle X \rangle$, there is a compact G-convex subset L_N of X with L_N containing N such that $\bigcap_{x \in L_N} g^c(x) \subset K$, and

(4) $\bigcup_{x \in X} g(x) = X$.

Then there is a B ∈ <X> such that $\Gamma_B \cap \bigcap_{x \in \Gamma_B} f^{-1}(x) \neq \phi$. In particular, f has a fixed point.

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新 **S-KKM** 映射的全交集定理及其應用

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摘　要

交集理論在非線性分析中，被廣泛討論及運用著。本篇文章裡，我們將在G-凸空間裡討論新S-KKM映射的交集理論。我們也討論了樊瓛型的配對理論，並討論集合值映射之定點的存在性作為它的應用。

關鍵字：凸空間，G-凸空間，G-凸集，Γ-有限閉集，新S-KKM映射，新G-KKM映射，配對理論，交集理論，定點。